Beyond Interference Avoidance: On Transparent Coexistence for Multi-hop Secondary CR Networks

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***Abstract*—This paper explores the so-called "transparent coexistence" paradigm for spectrum sharing between primary and secondary nodes in a multi-hop network environment. Although such paradigm has been studied in the information theory and communications communities, it is not well understood in the wireless networking community, particularly for multi-hop networks. Under this paradigm, a secondary network is allowed to use the same spectrum simultaneously with the primary network as long as their activities are "transparent" (or "invisible") to the primary network. Such transparency can be accomplished through a systematic interference cancellation (IC) by the secondary nodes without any impact on the primary network. This paper offers an in-depth study of this paradigm in a multi-hop network environment and addresses issues such as channel selection, IC to/from primary network, and IC within the secondary network. Through a rigorous modeling and formulation, we develop an optimization problem under this paradigm with the objective of maximizing secondary user's throughput. Through simulation results, we show that such paradigm offers significant improvement to a multi-hop network in terms of spectrum efficiency and throughput performance as compared to the prevailing interference-avoidance paradigm.**

I. INTRODUCTION

The current prevailing spectrum-sharing paradigm employed by the wireless networking community is that secondary nodes (typically equipped with cognitive radios (CRs)) are allowed to use a spectrum channel allocated to primary nodes only when the channel is not currently used by the primary nodes [1], [5], [21]. The rationale behind this paradigm is that per FCC requirements, secondary nodes should not produce interference that may be harmful to the primary nodes. In this paper, we call this *interference-avoidance* paradigm.[1] Under this paradigm, the wireless networking research community has invested significant research efforts in algorithm design and protocol implementation to optimize secondary CR users' performance while making sure that their spectrum usage will not interfere with the primary users.

On the other hand, in the information theory community, there is a strong interest in exploring information theoretic limit of CR [6]. In particular, researchers have been exploring the potential of *simultaneous activation* of a secondary network with the primary network. Here, secondary nodes are allowed to be active as long as they can cancel their interference to the primary nodes in such a way that the primary nodes do not feel the presence of the secondary nodes. In other words, activities by the secondary nodes are made in a *transparent* (or "invisible") way to primary nodes. We call this *transparent-coexistence* paradigm in this paper.[2] Under this paradigm, secondary nodes are assumed to have powerful physical layer capabilities to handle interference cancellation (IC). Further, the burden of IC should solely rest upon the secondary nodes so as to be truly "transparent" (or "invisible") to primary nodes. As expected, such a paradigm has the potential of offering much greater spectrum efficiency and network capacity than those under the interference-avoidance paradigm.

Although the idea of the transparent-coexistence paradigm has been explored to some extent in the information theory community, results from the information theory and communications communities have mainly limited to very simple network settings, e.g., several nodes or link pairs, all in *single-hop* communications [2], [7], [10], [19], [20]. Extending the transparent-coexistence paradigm to a *multi-hop* wireless network is not trivial. As we shall describe in detail in Section III, there are a number of significant challenges, such as channel selection, IC by secondary nodes to *and* from primary nodes, and IC by secondary nodes within the secondary network. Due to these challenges, there remain significant technical barriers to bring the idea of the transparent-coexistence paradigm to reality in a *multi-hop* network environment.

The goal of this paper is to make a fundamental advance in the transparent-coexistence paradigm so as to enhance access to the radio spectrum for multi-hop secondary CR networks. We aim to address the technical barriers in a *multi-hop wireless network* environment. We hope this effort will make a timely contribution to the wireless networking community so that this new paradigm can eventually evolve into the prevailing paradigm for CR research in the wireless networking community.

For canceling interference, we assume that each secondary node is equipped with multiple transmit/receive antennas (MIMO). Under the transparent-coexistence paradigm, we offer a systematic modeling of channel selection, IC between primary and secondary nodes, and IC within the secondary network. A key requirement in our model is that all IC burden should solely rest upon the secondary nodes. Based on our model, we consider a throughput optimization problem with the objective being maximizing the minimum session throughput in the multi-hop secondary network. Through simulation results, we demonstrate how the

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[1]This is also called "interweave" paradigm in [6].

[2]This is also called "underlay" paradigm in [6].

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transparent-coexistence paradigm can offer much improved spectrum efficiency and throughput performance than the current interference-avoidance paradigm.

The remainder of this paper is organized as follows. In Section II, we offer necessary background on MIMO and its DoF-based link model. Section III describes the scope of this paper and key challenges in employing transparent-coexistence paradigm in a multi-hop network environment. In Section IV, we present a mathematical model for the transparent-coexistence paradigm where both the primary and secondary networks are multi-hop. Based on this model, we also present a problem formulation for a throughput maximization problem in the secondary network. Section V presents simulation results and Section VI concludes this paper.

II. Background and Motivation

In this section, we give a brief review of MIMO's spatial multiplexing (SM) and IC capabilities [3], [9], [16], [17]. Other MIMO capabilities such as spatial diversity [22] are not explored in this paper and will be considered in our future work.

A simplified representation of MIMO model can be built upon the so-called degree-of-freedom (DoF) concept [17]. Simply put, the total number of DoFs at a node (no more than the number of antenna elements) represents the available resources at the node. A DoF can be used for either data transmission/reception or IC. Typically, transmitting one data stream requires one DoF at the transmitter and one DoF at a receiver. SM refers to the scenario where multiple DoFs are used to transmit multiple data streams, thus substantially increasing data throughput between two nodes. On the other hand, IC refers to a node's capability to use some of its DoFs to cancel interference, either as a transmitter or as a receiver. Depending on whether IC is done at a transmitter or receiver, the number of required DoF consumption may be different.

- If a transmitter is to cancel its interference to an unintended receiver, the number of DoFs required at this transmitter is equal to the number of data streams (or DoFs) that the unintended receiver is trying to receive from another transmitter.
- If a receiver is to cancel the interference from an interfering transmitter, the number of DoFs required at this receiver is equal to the number of data streams (or DoFs) that the interfering transmitter is trying to transmit to another receiver.

At any node, the sum of DoFs used for SM and IC cannot exceed the total number of DoFs at the node.

A MIMO node's ability to use a subset of its DoFs to cancel interference while to use the remaining subset of DoFs for data transmission allows the possibility of simultaneous activation of the secondary nodes with the primary nodes. Per FCC ruling, the operation of the secondary network should not impose any noticeable interference to the primary network. We use a simple example to illustrate this point. In Fig. 1, suppose T_p and R_p are a pair of transmitting and receiving nodes in the primary network, while T_s and R_s are a pair of transmitting

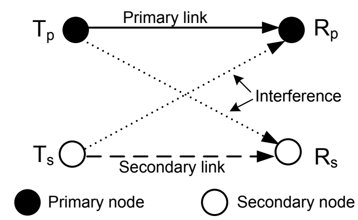

Fig. 1. A simple example illustrating the benefits of using MIMO to allow simultaneous activation of primary and secondary nodes.

and receiving nodes in the secondary network. Assume that all nodes share the same channel. Suppose T_p is transmitting 1 data stream to R_p. Under the interference-avoidance paradigm, a secondary transmitter (e.g., T_s) is prohibited from transmitting on the same channel in the neighborhood of R_p. However, when MIMO is employed on the secondary nodes, simultaneous transmission can be achieved. Assume secondary nodes T_s and R_s are each equipped with 4 antennas (with DoFs being 4). We can have T_s use 1 of its DoFs to cancel its interference to R_p so that R_p can receive its 1 data stream correctly from T_p. At node R_s, we can use 1 of its DoFs to cancel interference from T_p. After IC, both T_s and R_s still have 3 DoFs remaining, which can be used for SM of 3 data stream from T_s to R_s.

It is important to realize that we strive to put all IC burden on the secondary nodes side. Specifically, the transmitter of a secondary node needs to cancel its interference to all neighboring primary receiving nodes who are interfered by this secondary transmitter; the receiver of a secondary node needs to cancel interference from all neighboring primary transmitting nodes who interfere with this secondary receiver. To achieve transparency to primary nodes, it is important for the secondary nodes to have accurate channel state information (CSI). The problem here is: how can a secondary node obtain the CSI between itself and its neighboring primary nodes while remaining transparent to primary nodes? We propose the following solution to resolve this problem. For each primary node, it typically sends out a pilot sequence (training sequence) to its neighboring primary nodes such that those primary nodes can estimate the CSI for communication. This is the practice for current cellular networks and we assume such a mechanism is available for a primary network. Then, the secondary nodes can *overhear* the pilot sequence signal from the primary node while staying transparent. Suppose the pilot sequence from the primary nodes is publicly available (as in cellular networks) and are thus known to the secondary nodes. Then the secondary nodes can use this information and the actual received pilot sequence signal from the primary node for channel estimation. Based on the reciprocity property of a wireless channel [15], the estimated CSI can also be used as CSIT (channel state information at transmitter side). Therefore, a secondary node can obtain complete CSI between itself and a primary node.

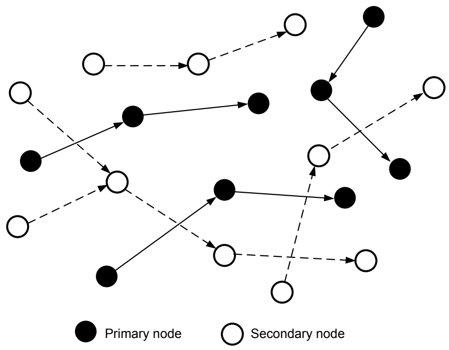

Fig. 2. A multi-hop secondary network co-located with a multi-hop primary network.

III. Problem Scope

Although the new coexistence paradigm has been explored at the physical layer, its application to a multi-hop network environment is far from trivial. Consider a primary multi-hop ad hoc network $\mathcal{P}$ shown in Fig. 2, which is co-located with a secondary multi-hop network $\mathcal{S}$ in the same geographical region. Suppose that there is a set of channels $\mathcal{B}$ available to the primary network. The primary nodes can use this set of frequency channels freely as if they were the only nodes in the network. The primary nodes do not need to be MIMO-capable. The secondary nodes, however, are allowed to use a channel in $\mathcal{B}$ only if their interference to the primary nodes are canceled properly, with complete transparency to the primary nodes. As discussed, the secondary nodes are assumed to be equipped with MIMO. In this context, we have a number of challenges for the secondary network.

- **Channel Selection** In a secondary network, an intermediate relay node is both a transmitter and a receiver. Due to half-duplex, a node cannot transmit and receive on the same channel at the same time. Therefore, scheduling (either in time slot or channel) is needed. In this paper, we assume scheduling is performed in the form of channel assignment. Therefore, a secondary relay node needs to select different channels for transmission and reception. Note that scheduling transmission and reception of a secondary node will lead to a different interference relationship among the primary and secondary nodes in the network. This brings in considerable complexity to the mathematical modeling of interference relationship.
- **IC to/from Primary Network** The essence of this challenge was illustrated in the example in Fig. 1 in the last section. Simply put, a secondary transmitter needs to cancel its interference to its neighboring primary receivers while a secondary receiver needs to cancel the interference from its neighboring primary transmitters. Such challenge magnifies when the secondary network is a multi-hop network.
- **IC within Secondary Network** In addition to interference between primary and secondary nodes, interference from a secondary node may also interfere another secondary node within their own network. Such interference must also be canceled properly (either by a secondary transmitter or the secondary receiver that is being interfered with) to ensure successful data communications inside the secondary network. Resource allocation to account for such IC is clearly not a trivial problem.

It is important to realize that the above three challenges are not independent, but rather deeply intertwined with each other. In particular, channel selection at a secondary node is directly tied to the interference relationship between primary and secondary nodes as well as interference among the secondary nodes within each channel. Further, the combined channel resource and total DoFs at each node determine a complete resource space in the network: an optimal DoF allocation and channel selection at each secondary node for both IC to/from the primary nodes and within the secondary nodes are critical to achieve the desired network performance objective. A modeling and formulation of transparent-coexistence paradigm would call for a joint consideration of all these components.

IV. Modeling and Formulation

In this section, we develop a mathematical model for the transparent-coexistence paradigm where a multi-hop secondary network shares the same spectrum as a primary network (see Fig. 2).

A. Mathematical Modeling

Referring to Fig. 2, we consider a secondary multi-hop network consisting of a set of nodes $\mathcal{S}$, that is co-located with a primary multi-hop network consisting of a set of nodes $\mathcal{P}$. Suppose that there is a set of channels $\mathcal{B}$ available to the primary network. For the primary network, there is no special node requirement and we assume that each primary node is a traditional single-antenna node. A primary node may transmit and receive on the same channel but in different time slot or transmit and receive on different channels. We consider the latter in this paper. Consider a set of multi-hop sessions $\tilde{\mathcal{F}}$ among the primary nodes. For a given routing for each session, denote $\tilde{\mathcal{L}}$ the set of active links in the primary network (shown in solid arrow lines in Fig. 2). Denote $\tilde{z}^b(\tilde{l})$ as the number of data streams over primary link $\tilde{l} \in \tilde{\mathcal{L}}$ on channel b. Then due to single antenna on each primary node, $\tilde{z}^b(\tilde{l}) = 1$ if link $\tilde{l}$ is active on channel b and 0 otherwise.

For the secondary network, we assume MIMO capability at each node. Denote A_i as the number of antennas on a secondary node $i \in \mathcal{S}$. Suppose there is a set of multi-hop sessions $\mathcal{F}$ in $\mathcal{S}$. For a given routing for each session, denote $\mathcal{L}$ as the set of secondary links (shown in dashed arrow line in Fig. 2). Denote $r(f)$ as the rate of session $f \in \mathcal{F}$. A general goal of throughput maximization is to maximize a function of $r(f)$, $f \in \mathcal{F}$.

Channel Selection. To model channel use behavior at a secondary node for transmission or reception, we denote x_i^b and y_i^b ($i \in \mathcal{S}$ and $b \in \mathcal{B}$) as whether node i selects channel b for transmission or reception, respectively. We have

$$x_i^b = \begin{cases} 1 & \text{if node } i \text{ uses channel } b \text{ for transmission;} \\ 0 & \text{otherwise.} \end{cases}$$

$$y_i^b = \begin{cases} 1 & \text{if node } i \text{ uses channel } b \text{ for reception;} \\ 0 & \text{otherwise.} \end{cases}$$

To consider half-duplex (a node cannot transmit and receive on the same channel at the same time), we have the following constraint on x_i^b and y_i^b:

$$x_i^b + y_i^b \le 1 \qquad (i \in \mathcal{S}, b \in \mathcal{B}). \tag{1}$$

Node Ordering for IC. Recall that the secondary network is solely responsible for IC to/from the primary network (as well as IC within itself). For IC, it has been shown in [14] that a node ordering can be employed to avoid unnecessary duplication in IC (and thus waste of DoF resources) while ensuring feasibility of a solution. By following an ordering among the secondary nodes, the IC to/from the primary network and IC within the secondary network can be performed as follows [14].

- If a secondary node is transmitting, then it has to allocate DoFs to null its interference to neighboring primary receivers. For IC within the secondary network, this secondary transmitter is only necessary to ensure that it does not interfere with neighboring secondary receivers that are before itself in the ordered secondary node list. This secondary transmitter does not need to expend DoFs to null its interference to those secondary receivers that are after itself in the ordered node list. Interference from this secondary transmitter to those secondary receivers (that are after this node in the ordered list) will be suppressed by those secondary receivers later.
- If a secondary node is receiving, then it has to allocate DoFs to null those interference from neighboring primary transmitters. For IC within the secondary network, this secondary receiver only needs to suppress interference from those neighboring secondary transmitters that are before itself in the ordered secondary node list. This secondary receiver does not need to concern with those interfering secondary transmitters that are after itself in the ordered node list. Interference from those secondary transmitters (that are after this secondary receiver in the node list) will be suppressed by those nodes later.

We point out that such an node ordering approach for DoF allocation is the most efficient approach among all existing DoF models that can guarantee feasibility. As pointed out in [14], an optimal ordering of secondary nodes can be found by inserting a formulation of the ordering relationship into the specific optimization problem.

Denote π^b as an ordered list of the secondary nodes in the network on channel $b \in \mathcal{B}$, and denote π_i^b as the position of node $i \in \mathcal{S}$ in π^b. Therefore, $1 \le \pi_i^b \le S$, where $S = |\mathcal{S}|$. For example, if $\pi_i^b = 3$, then it means that node i is the third node in the list π^b.

To model the relative ordering between any two secondary nodes i and j in π^b, we use a binary variable $\theta_{j,i}^b$ and define it as follows:

$$\theta_{j,i}^b = \begin{cases} 1 & \text{if node } j \text{ is before node } i \text{ in } \pi^b \text{ on channel } b; \\ 0 & \text{otherwise.} \end{cases}$$

It was shown in [14] that the following relationships hold.

$$\pi_i^b - S\cdot\theta_{j,i}^b + 1 \le \pi_j^b \le \pi_i^b - S\cdot\theta_{j,i}^b + S - 1 \ , \quad (i, j \in \mathcal{S}, b \in \mathcal{B}). \tag{2}$$

DoF Allocation at a Secondary Transmitter. At a secondary transmitter, it needs to expend DoFs for SM, IC to primary receivers, and IC to other secondary receivers.

- For SM, denote $z^b(l)$ and $\mathcal{L}_{i,\text{Out}}^b$ as the number of data streams over link $l \in \mathcal{L}$ and the set of outgoing links from secondary node i on channel b. Then the number of DoFs at secondary node $i \in \mathcal{S}$ for SM on channel b is $\sum_{l \in \mathcal{L}_{i,\text{Out}}^b} z^b(l)$.
- For IC to primary receivers, recall $\tilde{z}^b(\tilde{l})$ is the number of data streams over primary link $\tilde{l}$ on channel b. For a primary node $p \in \mathcal{P}$, denote $\tilde{\mathcal{L}}_{p,\text{In}}^b$ as the set of incoming primary links on channel b. Denote $\tilde{\mathcal{I}}_i$ as the set of neighboring primary nodes that are located within the interference range of secondary transmitter i. Then at node i, the number of DoFs required for IC to primary receivers is $\left(\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}_{p,\text{In}}^b} \tilde{z}^b(\tilde{l})\right)$ on channel b.
- For IC to secondary receivers, as discussed earlier, a secondary transmitter i only needs to cancel its interference to those nodes that are before itself in the ordered list. For a secondary node $j \in \mathcal{S}$, denote $\mathcal{L}_{j,\text{In}}^b$ as the set of incoming secondary links. Denote $\mathcal{I}_i$ as the set of neighboring secondary nodes that are located within the interference range of secondary transmitter i. Then at node i, the number of DoFs required for IC to secondary receivers is $\sum_{j \in \mathcal{I}_i} \left(\theta_{j,i}^b \cdot \sum_{k \in \mathcal{L}_{j,\text{In}}^b}^{\text{Tx}(k) \ne i} z^b(k)\right)$ on channel b, and $\text{Tx}(k)$ represents the transmitter of link k.

Putting all three DoF consumptions together at a secondary transmitter i, we have the following constraints:

$$x_i^b \le \sum_{l \in \mathcal{L}_{i,\text{Out}}^b} z^b(l) + \left[\left(\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}_{p,\text{In}}^b} \tilde{z}^b(\tilde{l})\right) + \sum_{j \in \mathcal{I}_i} \left(\theta_{j,i}^b \cdot \sum_{k \in \mathcal{L}_{j,\text{In}}^b}^{\text{Tx}(k) \ne i} z^b(k)\right)\right] \cdot x_i^b \le x_i^b A_i \ , \tag{3}$$

which means that if node i is transmitting, the DoF consumptions cannot exceed the total number of DoFs at node i; if node i is not transmitting, there is no DoF consumption for transmissions, and $\sum_{l \in \mathcal{L}_{i,\text{Out}}^b} z^b(l) = 0$. By introducing a large constant M, which is an upper bound of $\left[\left(\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}_{p,\text{In}}^b} \tilde{z}^b(\tilde{l})\right) + \sum_{j \in \mathcal{I}_i} \left(\theta_{j,i}^b \cdot \sum_{k \in \mathcal{L}_{j,\text{In}}^b}^{\text{Tx}(k) \ne i} z^b(k)\right)\right]$ (e.g., $M = \sum_{j \in \mathcal{I}_i} A_j + \sum_{p \in \tilde{I}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}_{p,\text{In}}^b} \tilde{z}^b(\tilde{l})$), we can use the following two sets of constraints to replace (3):

$$x_i^b \le \sum_{l \in \mathcal{L}_{i,\text{Out}}^b} z^b(l) + \left(\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}_{p,\text{In}}^b} \tilde{z}^b(\tilde{l})\right) + \sum_{j \in \mathcal{I}_i} \left(\theta_{j,i}^b \cdot \sum_{k \in \mathcal{L}_{j,\text{In}}^b}^{\text{Tx}(k) \ne i} z^b(k)\right) \le A_i x_i^b + (1 - x_i^b)M \ , \tag{4}$$

$$\sum_{l \in \mathcal{L}^b_{i,\text{Out}}} z^b(l) \le x^b_i A_i \ . \quad (5)$$

We can see that when node i is transmitting (i.e., $x^b_i = 1$), (4) becomes (3) and (5) holds trivially; if node i is not transmitting (i.e., $x^b_i = 0$), (5) and (3) are equivalent, which is $\sum_{l \in \mathcal{L}^b_{i,\text{Out}}} z^b(l) = 0$, and (4) holds trivially.

Since (4) has a nonlinear term $\left(\theta^b_{j,i} \cdot \sum_{k \in \mathcal{L}^b_{j,\text{In}}}^{\text{Tx}(k)\neq i} z^b(k)\right)$, we can use *Reformulation-Linearization Technique* (RLT) [13] to reformulate this nonlinear term as several linear terms. We define a new variable $\lambda^b_{j,i}$ as follows:

$$\lambda^b_{j,i} = \theta^b_{j,i} \cdot \sum_{k \in \mathcal{L}^b_{j,\text{In}}}^{\text{Tx}(k)\neq i} z^b(k) \ , \qquad (i \in \mathcal{S}, j \in \mathcal{I}_i, b \in \mathcal{B}).$$

For binary variable $\theta^b_{j,i}$, we have the following related constraints: $\theta^b_{j,i} \ge 0, (1 - \theta^b_{j,i}) \ge 0$. For $\sum_{k \in \mathcal{L}^b_{j,\text{In}}}^{\text{Tx}(k)\neq i} z^b(k)$, we have $\sum_{k \in \mathcal{L}^b_{j,\text{In}}}^{\text{Tx}(k)\neq i} z^b(k) \ge 0$ and $A_j - \sum_{k \in \mathcal{L}^b_{j,\text{In}}}^{\text{Tx}(k)\neq i} z^b(k) \ge 0$. We can multiply each constraint involving $\theta^b_{j,i}$ by one of the two constraints involving $\sum_{k \in \mathcal{L}^b_{j,\text{In}}}^{\text{Tx}(k)\neq i} z^b(k)$, and replacing the product term $\left(\theta^b_{j,i} \cdot \sum_{k \in \mathcal{L}^b_{j,\text{In}}}^{\text{Tx}(k)\neq i} z^b(k)\right)$ with a new variable $\lambda^b_{j,i}$. Then (4) can be replaced by the following linear constraints.

$$\begin{aligned} x^b_i &\le \sum_{l \in \mathcal{L}^b_{i,\text{Out}}} z^b(l) + \left(\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}^b_{p,\text{In}}} \tilde{z}^b(\tilde{l})\right) + \sum_{j \in \mathcal{I}_i} \lambda^b_{j,i} \\ &\le A_i x^b_i + (1 - x^b_i)M \quad (i \in \mathcal{S}, b \in \mathcal{B}), \end{aligned} \quad (6)$$

$$\lambda^b_{j,i} \ge 0 \qquad (i \in \mathcal{S}, j \in \mathcal{I}_i, b \in \mathcal{B}), \quad (7)$$

$$\lambda^b_{j,i} \le \sum_{k \in \mathcal{L}^b_{j,\text{In}}}^{\text{Tx}(k)\neq i} z^b(k) \qquad (i \in \mathcal{S}, j \in \mathcal{I}_i, b \in \mathcal{B}), \quad (8)$$

$$\lambda^b_{j,i} \le A_j \cdot \theta^b_{j,i} \qquad (i \in \mathcal{S}, j \in \mathcal{I}_i, b \in \mathcal{B}), \quad (9)$$

$$\lambda^b_{j,i} \ge A_j \cdot \theta^b_{j,i} - A_j + \sum_{k \in \mathcal{L}^b_{j,\text{In}}}^{\text{Tx}(k)\neq i} z^b(k) \qquad (i \in \mathcal{S}, j \in \mathcal{I}_i, b \in \mathcal{B}). \quad (10)$$

DoF Allocation at a Secondary Receiver. At a secondary receiver, it needs to expend DoFs for SM, for IC from primary transmitters, and for IC from other secondary transmitters. For a primary node $p \in \mathcal{P}$, denote $\tilde{\mathcal{L}}^b_{p,\text{Out}}$ as the set of outgoing primary links. Following the same token as our discussion for a secondary transmitter, we can put all DoF consumption at a secondary receiver as follows:

$$\begin{aligned} y^b_i \le & \sum_{k \in \mathcal{L}^b_{i,\text{In}}} z^b(k) + \left(\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}^b_{p,\text{Out}}} \tilde{z}^b(\tilde{l})\right) + \\ & \sum_{j \in \mathcal{I}_i} \left(\theta^b_{j,i} \cdot \sum_{l \in \mathcal{L}^b_{j,\text{Out}}}^{\text{Rx}(l)\neq i} z^b(l)\right) \le A_i y^b_i + (1 - y^b_i)N \ , \end{aligned} \quad (11)$$

$$\sum_{k \in \mathcal{L}^b_{i,\text{In}}} z^b(k) \le y^b_i A_i \ , \quad (12)$$

where $\sum_{k \in \mathcal{L}^b_{i,\text{In}}} z^b(k)$ represents the number of DoFs used for SM, $\left(\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}^b_{p,\text{Out}}} \tilde{z}^b(\tilde{l})\right)$ represents the number of DoFs used for suppressing interference from primary transmitters, and $\sum_{j \in \mathcal{I}_i} \left(\theta^b_{j,i} \cdot \sum_{l \in \mathcal{L}^b_{j,\text{Out}}}^{\text{Rx}(l)\neq i} z^b(l)\right)$ represents the number of DoFs consumed for canceling interference from other secondary transmitters, and N represents a large constant, and $\text{Rx}(l)$ represents the receiver of link l.

Again, we can use RLT to linearize the nonlinear term $\left(\theta^b_{j,i} \cdot \sum_{l \in \mathcal{L}^b_{j,\text{Out}}}^{\text{Rx}(l)\neq i} z^b(l)\right)$ in (11). Denote $\mu^b_{j,i}$ as $\left(\theta^b_{j,i} \cdot \sum_{l \in \mathcal{L}^b_{j,\text{Out}}}^{\text{Rx}(l)\neq i} z^b(l)\right)$. Then (11) can be replaced by the following linear constraints:

$$\begin{aligned} y^b_i &\le \sum_{k \in \mathcal{L}^b_{i,\text{In}}} z^b(k) + \left(\sum_{p \in \tilde{\mathcal{I}}_i} \sum_{\tilde{l} \in \tilde{\mathcal{L}}^b_{p,\text{Out}}} \tilde{z}^b(\tilde{l})\right) + \sum_{j \in \mathcal{I}_i} \mu^b_{j,i} \\ &\le A_i y^b_i + (1 - y^b_i)N \qquad (i \in \mathcal{S}, b \in \mathcal{B}), \end{aligned} \quad (13)$$

$$\mu^b_{j,i} \ge 0 \qquad (i \in \mathcal{S}, j \in \mathcal{I}_i, b \in \mathcal{B}), \quad (14)$$

$$\mu^b_{j,i} \le \sum_{l \in \mathcal{L}^b_{j,\text{Out}}}^{\text{Rx}(l)\neq i} z^b(l) \qquad (i \in \mathcal{S}, j \in \mathcal{I}_i, b \in \mathcal{B}), \quad (15)$$

$$\mu^b_{j,i} \le A_j \cdot \theta^b_{j,i} \qquad (i \in \mathcal{S}, j \in \mathcal{I}_i, b \in \mathcal{B}), \quad (16)$$

$$\mu^b_{j,i} \ge A_j \cdot \theta^b_{j,i} - A_j + \sum_{l \in \mathcal{L}^b_{j,\text{Out}}}^{\text{Rx}(l)\neq i} z^b(l) \qquad (i \in \mathcal{S}, j \in \mathcal{I}_i, b \in \mathcal{B}). \quad (17)$$

Link Capacity Constraint. For link $l \in \mathcal{L}$, we have the following link capacity constraint:

$$\sum_{f \in \mathcal{F}}^{f \text{ traversing } l} r(f) \le c \cdot \sum_{b \in \mathcal{B}} z^b(l) \qquad (l \in \mathcal{L}), \quad (18)$$

where c is the data rate carried by a data stream.

B. Formulation

Based on the above mathematical model, various problems can be formulated. In this paper, we study a throughput optimization problem with the objective of maximizing the minimum session rate among all secondary sessions. The optimization problem can be written as follows:

OPT
max $r_{\min}$
s.t $r_{\min} \le r(f) \qquad (f \in \mathcal{F})$;
Half duplex constraints: (1);
Node ordering constraints: (2);
Transmitter DoF constraints: (5)–(10);
Receiver DoF constraints: (12)–(17);
Link capacity constraints: (18).

In this formulation, $r_{\min}, r(f), x^b_i, y^b_i, z^b(l), \pi^b_i, \lambda^b_{j,i}, \mu^b_{j,i}$ and $\theta^b_{j,i}$ are optimization variables, and $A_i, M, N, \tilde{z}^b(\tilde{l})$ and c are given constants. This optimization problem is in the form of a mixed-integer linear program (MILP). Although the theoretical

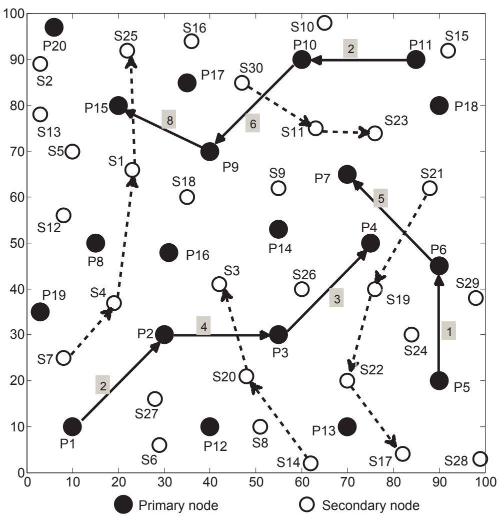

Fig. 3. Active sessions in the primary and secondary networks.

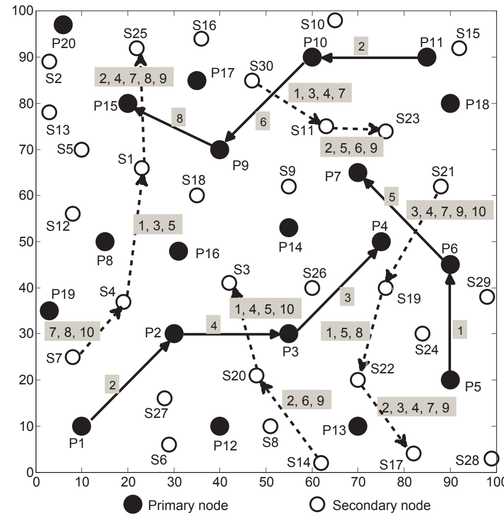

Fig. 4. Channel allocation on each link for the secondary sessions. Channel allocation on each link for the primary sessions are given *a priori*.

worst-case complexity to a general MILP problem is exponential [4], [11], there exist highly efficient optimal/approximation algorithms (e.g., branch-and-bound with cutting planes [12]) and heuristics (e.g., sequential fixing algorithm [8], [18]) to solve it.

V. Performance Evaluation

The goal of this section is twofold. First, we want to use numerical results to demonstrate how a secondary network can operate simultaneously with the primary network while being transparent to the primary network. Second, we will show the tremendous benefits in terms of throughput gain under the transparent-coexistent paradigm.

A. An Example

Consider a 20-node primary network and a 30-node secondary network randomly deployed in the same 100×100 area (see Fig. 3). For the ease of scalability and generality, we normalize all units for distance, bandwidth, and throughput with appropriate dimensions. As discussed in Section IV-A, the primary nodes are traditional single-antenna device while the secondary nodes are equipped with MIMO. We assume there are four antennas for transmission or reception on each secondary node. Further, we assume all nodes' transmission range and interference range are 30 and 50, respectively on all channels. There are $|\mathcal{B}| = 10$ channels available in the network. For simplicity, we assume the achievable rate of one DoF on a channel is 1 unit. In this case study, we assume there are three active sessions in the primary network and four active sessions in the secondary network. For simplicity, we assume that minimum-hop routing is used for each primary and secondary session. Further, the channel allocation on each hop for a primary session is known *a priori* (see Fig. 3).

For this network setting for primary and secondary networks, the obtained objective value is 7. The channel allocation on each link for each secondary session is shown in Fig. 4. The details of DoFs used for SM on each channel at each link are shown in Table I. The achievable rate (i.e., total number of DoFs used for SM) on a link is also shown in this table.

To see how links in the primary and secondary networks can be active on the same channel at the same time, consider channel 2 in Fig. 4. For channel 2, it is active on $P_1 \to P_2$ and $P_{11} \to P_{10}$ in the primary network and $S_{14} \to S_{20}$, $S_{22} \to S_{17}$, $S_1 \to S_{25}$, and $S_{11} \to S_{23}$ in the secondary network. The interference relationships among these 6 links are shown in Fig. 5, where the dotted arrow lines show the interference relationships among them. The two primary links $P_1 \to P_2$ and $P_{11} \to P_{10}$ do not interfere with each other as the receiver of each link is outside the interference range of the other link's transmitter. But each of these two primary links are within the interference range of its neighboring secondary links. Now consider link $P_1 \to P_2$.

- To cancel interference from secondary nodes (S_1, S_{14}, and S_{22}) to primary node P_2, secondary transmitter S_1, S_{14}, and S_{22} use one DoF to cancel their interference to primary receiver P_2. Consequently, the transmissions on $S_1 \to S_{25}$, $S_{14} \to S_{20}$, and $S_{22} \to S_{17}$ will be transparent to primary node P_2.
- To cancel interference from primary node P_1 to secondary receiving node S_{20}, S_{20} uses one DoF to cancel this interference.
- Among the secondary links, $S_{14} \to S_{20}$ and $S_{22} \to S_{17}$ interfere with each other since the receiver of each link falls within the interference range of the transmitter of the other link. To cancel its interference to S_{17}, transmitter S_{14} uses one DoF to cancel this interference. On the other hand, to cancel the interference from S_{22}, receiver S_{20} uses one DoF to cancel this interference. After IC, nodes S_{14} and S_{20} can use the remaining 2 DoFs for SM (on both transmitter and receiver sides) while nodes S_{22} and S_{17} can only use 1 DoF for SM to meet IC constraints

TABLE I
CHANNEL ALLOCATION ON EACH LINK, DOF ALLOCATION ON EACH CHANNEL FOR SM, AND ACHIEVABLE DATA STREAMS ON EACH LINK FOR THE SECONDARY SESSIONS.

Session	Link	Channel Allocation	DoF for SM	Achievable Data Streams
1	$S_7 \longrightarrow S_4$	7	3	7
		8	2	
		10	2	
	$S_4 \longrightarrow S_1$	1	2	7
		3	2	
		5	3	
	$S_1 \longrightarrow S_{25}$	2	1	7
		4	1	
		7	1	
		8	1	
		9	3	
2	$S_{21} \longrightarrow S_{19}$	3	2	7
		4	1	
		7	1	
		9	1	
		10	2	
	$S_{19} \longrightarrow S_{22}$	1	1	7
		5	2	
		8	4	
	$S_{22} \longrightarrow S_{17}$	2	1	7
		3	1	
		4	1	
		7	3	
		9	1	
3	$S_{14} \longrightarrow S_{20}$	2	2	7
		6	4	
		9	1	
	$S_{20} \longrightarrow S_3$	1	2	7
		4	2	
		5	1	
		10	2	
4	$S_{30} \longrightarrow S_{11}$	1	2	7
		3	1	
		4	1	
		7	3	
	$S_{11} \longrightarrow S_{23}$	2	2	7
		5	1	
		6	3	
		9	1	

(6) and (13).

The discussion for primary link $P_{11} \to P_{10}$ is similar and is omitted to conserve space. In addition to channel 2, other channels that exhibit transparent-coexistence between primary and secondary links include channels 1, 3, 4, 5, 6 and 8.

B. Comparison to Interference-Avoidance Paradigm

To see the benefits of the transparent-coexistence paradigm, we compare it to the prevailing interference-avoidance paradigm. Under the interference-avoidance paradigm, a secondary node is not allowed to transmit (receive) on a channel when a nearby primary receiver (transmitter) is using the same channel. Therefore, the set of available channels that can be used for secondary nodes is smaller. The problem formulation for this paradigm is simpler (although somewhat similar) than OPT. In particular, we can remove the second term ($\sum_{p\in\tilde{\mathcal{I}}_i}\sum_{\tilde{l}\in\tilde{\mathcal{L}}^b_{p,\mathrm{In}}}\tilde{z}^b(\tilde{l})$ and $\sum_{p\in\tilde{\mathcal{I}}_i}\sum_{\tilde{l}\in\tilde{\mathcal{L}}^b_{p,\mathrm{Out}}}\tilde{z}^b(\tilde{l})$) in constraints (6) and (13) in OPT that are used for secondary nodes to cancel interference to/from the primary nodes.

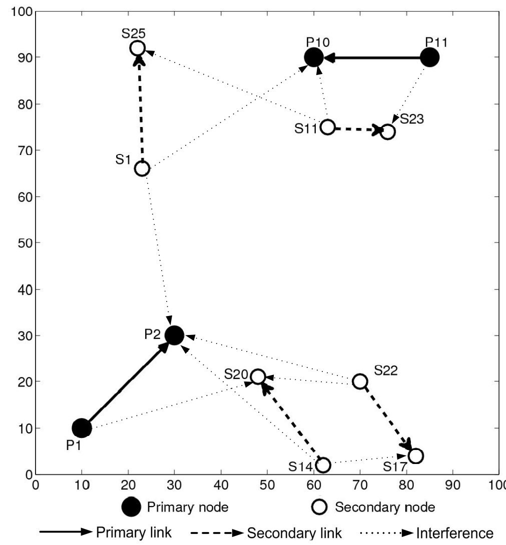

Fig. 5. Illustration of interference relationships among the primary and secondary links on channel 2 in the case study.

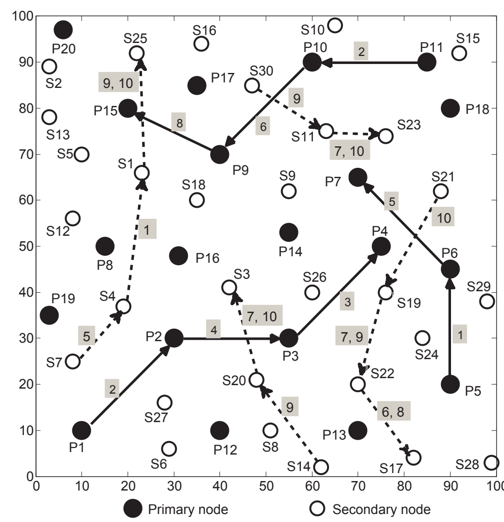

Fig. 6. Channel allocation on each link under the interference-avoidance paradigm.

Following the same setting as in the case study in Section V-A, we solve the above optimization problem under the interference-avoidance paradigm. The obtained objective value is 3 (compared to 7 in Section V-A). The channel allocation on each link for each secondary session is shown in Fig. 6. Comparing Figs. 4 with 6, we find that the set of channels used on each secondary link under interference-avoidance paradigm is smaller than that under transparent-coexistence paradigm.

C. Additional Results

Following the same token as for the case study in the last section, we generate 50 additional instances of 20-node primary network and 30-node secondary network. For each instance, we randomly generate primary and secondary sessions, and compare the objective values obtained by the transparent-

TABLE II
ACHIEVED DATA STREAMS UNDER TRANSPARENT COEXISTENCE (TC) PARADIGM AND INTERFERENCE-AVOIDANCE PARADIGM FOR 50-NODE NETWORK INSTANCES.

Network Instance	TC	Interference Avoidance	Network Instance	TC	Interference Avoidance
1	8	2	26	7	1
2	7	2	27	6	2
3	4	2	28	5	2
4	4	0	29	5	0
5	13	4	30	7	2
6	5	2	31	10	4
7	4	2	32	10	5
8	11	5	33	4	2
9	3	0	34	7	0
10	3	0	35	5	0
11	6	2	36	8	4
12	7	0	37	6	0
13	5	1	38	5	0
14	13	4	39	9	4
15	7	2	40	4	0
16	7	0	41	6	1
17	12	4	42	8	4
18	9	4	43	3	2
19	6	0	44	4	0
20	6	2	45	5	2
21	11	6	46	6	0
22	4	2	47	8	4
23	7	4	48	4	2
24	6	0	49	5	0
25	5	3	50	8	4

coexistence paradigm and interference-avoidance paradigm. Table II shows the results from 50 network instances. We find that the achievable data streams under the transparent-coexistence paradigm are much higher than those under the interference-avoidance paradigm.

VI. CONCLUSIONS

The goal of this paper was to offer a systematic study of the transparent-coexistence paradigm in a multi-hop network environment. The main technical challenges are channel allocation, IC to/from primary network by the secondary network, and IC within the secondary network. Through mathematical modeling, problem formulation, and performance evaluation, we show that the transparent-coexistence paradigm offers significant improvement in spectrum efficiency and throughput performance over the existing prevailing interference-avoidance paradigm. The mathematical development and results from this paper will lay an important step stone to advance the transparent-coexistence paradigm in the wireless networking research community.

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